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Rendezvous Guidance of Lifting Aerospace Vehicles

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An explicit guidance method is described for space vehicles that 1) use atmospheric forces to permit maneuvers during ascent to orbit; 2) ascend directly to an orbital rendezvous; and 3) perform a hypersonic, in-atmosphere cruise before light-off of rocket-powered ascent to orbit. Hypersonic cruise capability will give the vehicle great flexibility to reach targets because it can, in effect, move its launch point relative to earth. The guidance system, however, must accept variations in position, heading, and time of rocket ignition and still must complete the rendezvous. This method can accept the moving-rocket ignition point because place and time of rendezvous are not specified in advance but depend upon the geocentric lead angle between vehicle and target at ignition. Guidance constraint of slope at injection allows reasonable variation in time of ignition, with near-minimum fuel penalty. Simulation results for a representative future lifting ascent vehicle show allowable variation in position at rocket ignition of 80 miles on either side of target orbit plane, with allowed time variation of about 3 min at any position. The vehicle can start farther from the target plane by making a maneuvering ascent in a different plane and then a dog-leg.

Introduction

The class of future aerospace vehicles that will make use of aerodynamic forces instead of minimizing their effects during ascent to orbit is appearing ever larger on the horizon. These vehicles may evolve from hypersonic aircraft that extend their region of operation to orbit either directly or with staging, or they may embody new design concepts. They have at various times been called recoverable or reusable boosters, or simply aerospace planes. A typical ascent for such a vehicle will begin with a hypersonic in-atmosphere cruise that brings the vehicle to the light-off point where rocket-powered ascent begins (Fig. 1). The vehicle can maneuver during the rocket-powered ascent in order to arrive at a cutoff point from which it will coast to its destination in orbit.

The reusable characteristic will give this type of vehicle economy that will motivate its development for future high-traffic-density missions such as logistic shuttles. Equally important, the in-atmosphere cruise capability and the maneuver during rocket ascent will give it considerable flexibility to reach any of a large number of orbital destinations directly from the same starting point. This will make the vehicle particularly suitable for quick reaction missions such as rescue and satellite inspection.

This paper describes a method for explicit guidance during the maneuvering rocket-powered ascent, for vehicles and missions having these characteristics: 1) use of aerodynamic lift to maneuver during ascent, 2) ascent directly to an orbital rendezvous, and 3) use of hypersonic cruise prior to the rocket-powered ascent. These vehicle-mission characteristics, shown in Fig. 1, have a strong impact on the guidance requirements.

When a vehicle is launched from a fixed site on earth to reach an orbital target, without a hypersonic cruise, fuel economy generally dictates that the launching wait until the earth's rotation brings the launch site close to the target orbit plane. If the target is not then in a favorable location in its orbit, the ascending vehicle will enter a parking or phasing orbit. If a direct ascent to the target is required, the launching may have to take place when the launch site is some distance away from the target orbit plane in order that the ascending vehicle will meet the target upon arrival in orbit; the velocity increment required to turn into the target orbit plane may be considerable. References 1–4 are among the papers that discuss rendezvous starting from earth-fixed launch sites.

A vehicle capable of hypersonic cruise before rocket ignition will generally direct its cruise path so as to have one component toward the target orbit plane and another in the direction of the target velocity in orbit. That is, prior to the rocket ascent, the vehicle will move its launch point or point

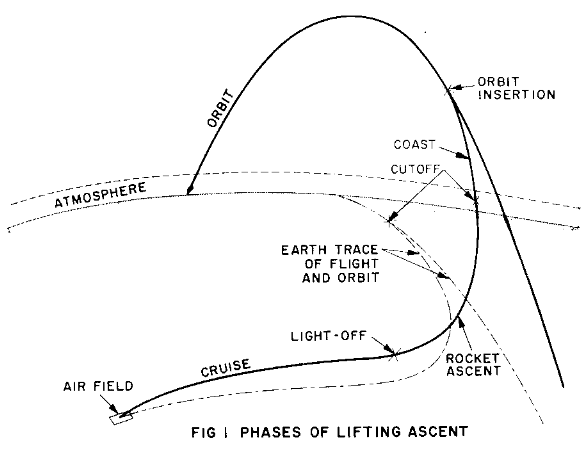

Fig. 1 Phases of lifting ascent.

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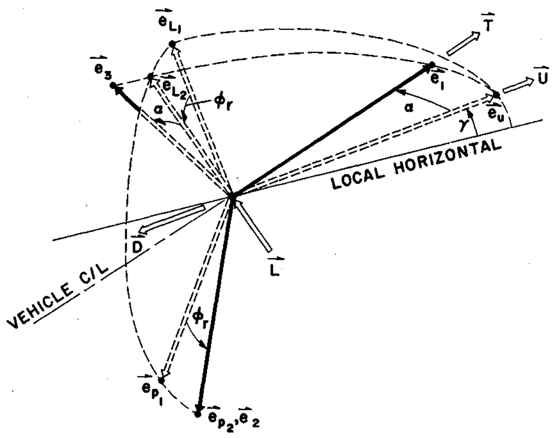

Fig. 2 Coordinate systems and forces.

of rocket ignition relative to the earth and will begin its ascent when it is in a more favorable location relative to the phase of the target in orbit.

This ability to move the launch point is a great advantage from the over-all mission standpoint. However, the precise prediction and control of the position and heading of the aircraft at fixed instants of time would be very difficult. To attempt to do this would jeopardize much of the potential flexibility of the vehicle. To realize the full potential of the concept, the guidance system should allow the vehicle position and velocity at the time of rocket ignition to be anywhere within its range of physical capability to reach the target. Explicit guidance should be used to insure rendezvous. No unclassified discussions of ascent guidance could be found in the literature for vehicles that use aerodynamic forces or that start their rocket-powered ascent from a point moving in the atmosphere.

This paper describes an explicit ascent guidance method that will meet these requirements. It consists of ascent and out-of-plane guidance equations that operate in parallel to define the vehicle's path. The out-of-plane guidance can accept a considerable variation in initial heading and position at any distance from the ascent plane from which the vehicle is capable of reaching the plane and aligning in it before thrust cutoff. The ascent guidance, meanwhile, defines an ascent path to cutoff which will place the vehicle in a coasting path to meet the target. The basis of this guidance scheme's ability to accept a moving-rocket ignition point is the fact that the place and time of rendezvous are not specified in advance; they depend upon the geocentric lead angle between the ascending vehicle and the target at rocket ignition and upon the ascent profile and the disturbances encountered. The guidance constraint that defines the coasting ascent ellipse is that the slope at arrival must equal that of the target orbit. This gives near-minimum fuel consumption and allows a reasonable variation in the time of rocket ignition at any given position by varying the geocentric range angle between the light-off point and rendezvous. An atmospheric form of cross-product steering permits explicit guidance to begin while the vehicle is still within the sensible atmosphere.

Equations of Motion

The equations of motion are integrated in rectangular, geocentric, inertial coordinates X, Y, Z. The vehicle ascends through the atmosphere in the XY plane. In the simulations reported later in this paper, the X, Y axes were oriented so that $Y = 0$ at rocket ignition.

The vehicle's position is

$$\mathbf{r} = \mathbf{i}X + \mathbf{j}Y + \mathbf{k}Z \tag{1}$$

and its inertial velocity is

$$\mathbf{V} = \dot{X}\mathbf{i} + \dot{Y}\mathbf{j} + \dot{Z}\mathbf{k} \tag{2}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors along X, Y, Z. The velocity relative to the rotating air mass is

$$\mathbf{U} = \mathbf{V} - \mathbf{\Omega} \times \mathbf{r} \tag{3}$$

where $\mathbf{\Omega}$ is the earth's angular velocity. The earth is assumed spherical with a central force field.

The vehicle's angle of attack is denoted by a and its roll angle by ϕ_r. It is assumed that, when a and ϕ_r are both zero, the vehicle centerline lies along $\mathbf{U}$, i.e., the vehicle makes coordinated turns by rotating the lift vector; yawing turns are not used.

Since aerodynamic forces are defined along and normal to $\mathbf{U}$, it is convenient to establish orthogonal coordinates described by the unit vectors $\mathbf{e}_u$, $\mathbf{e}_{L1}$, $\mathbf{e}_{p1}$ in direction defined by $\mathbf{U}$, $(\mathbf{U} \times \mathbf{r}) \times \mathbf{U}$, and $\mathbf{U} \times \mathbf{r}$, respectively, and another set $\mathbf{e}_u$, $\mathbf{e}_{L2}$, $\mathbf{e}_{P2}$ generated by rolling the first set about $\mathbf{U}$ through the angle ϕ_r. Figure 2 shows these and the pertinent forces. The equations of motion in vector form are

$$\Sigma F = m(d\mathbf{V}/dt) = L\mathbf{e}_{L2} - D\mathbf{e}_u + T(\mathbf{e}_u \cos a + \mathbf{e}_{L2} \sin a) - (\mu m/r^3)(X\mathbf{i} + Y\mathbf{j} + Z\mathbf{k}) \tag{4}$$

and the rectangular components of forces are

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \end{Bmatrix} = \begin{bmatrix} e_{ux} & e_{L1x} & e_{p1x} \\ e_{uy} & e_{L1y} & e_{p1y} \\ e_{uz} & e_{L1z} & e_{p1z} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi r & -\sin\phi r \\ 0 & \sin\phi r & \cos\phi r \end{bmatrix} \begin{Bmatrix} -D + T\cos a \\ L + T\sin a \\ 0 \end{Bmatrix} - \frac{\mu m}{r^3} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} \tag{5}$$

where e_{ux}, e_{uy}, e_{uz}, e_{L1x}, e_{L1y}, etc., are the direction cosines between the axes indicated by the subscripts.

Ascent Guidance

Powered ascent from the light-off point consists of a period during which the vehicle is guided along a predetermined profile followed by a period of explicit guidance to thrust cutoff. The object of the predetermined profile is to keep the vehicle clear of structural and thermal boundaries during acceleration through the denser atmosphere. It can take any of several forms, so long as it is economical in propellant and provides acceptable transition to explicit guidance. The particular profile chosen will depend upon the vehicle and the mission.

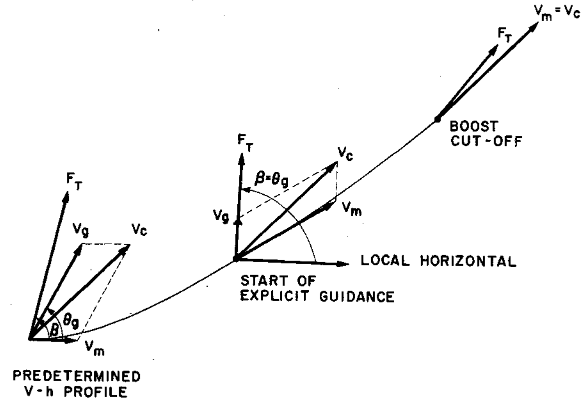

Fig. 3 Planar steering sequence, local vertical definition.

A velocity-altitude profile has been used in the study, defined by

$$U_r = f(h) \tag{6}$$

where U_r is the required value of the magnitude of U, and

$$h = |\mathbf{r}| - r_0 \tag{7}$$

where r_0 is the earth's radius.

The explicit guidance computation is carried on from the beginning of ascent. The guidance system continually computes the characteristic velocity $\mathbf{V}_c$ which would effect a rendezvous from the vehicle's present position, and also the velocity to be gained $\mathbf{V}_g$. The total measured force $\mathbf{F}$ (thrust plus lift plus drag) is also computed; the acceleration caused by $\mathbf{F}$ would be sensed by a set of onboard accelerometers. When the direction of $\mathbf{F}$ coincides with the direction of $\mathbf{V}_g$, as it will at some time during the predetermined portion of the ascent, steering command is transferred to explicit guidance, and the vehicle is thereafter steered in pitch so as to keep $\mathbf{F}$ aligned in the direction of $\mathbf{V}_g$ until $\mathbf{V}_g$ is reduced to zero, at which time thrust is terminated (Fig. 3).

Aligning the total measured force along $\mathbf{V}_g$ is a proper definition of in-atmosphere cross-product steering. As the vehicle leaves the sensible atmosphere, this reduces to aligning the thrust along $\mathbf{V}_g$

The guidance constraint on which the computation of characteristic velocity is based is that the slope of the ascent ellipse must equal the slope of the target orbit at the rendezvous point (Fig. 4). This places the ascending vehicle in a favorable state for insertion into the target orbit and results in near-minimum propellant consumption when both ascent and insertion are considered. The radial and tangential components V_{cr} and V_{ct} of the characteristic velocity are

$$V_{ct} = \left[\frac{\mu}{r}\left\{\frac{1 - \cos\phi_a}{1 - (r/r_T)\cos\phi_a - (r/r_T)m_a\sin\phi_a}\right\}\right]^{1/2} \tag{8}$$

$$V_{cr} = V_{ct}\left[\left\{1 - \frac{r}{r_T}\right\}\left\{\frac{1 - \cos\phi_a}{\sin\phi_a}\right\} - \frac{rm_a}{r_T}\right] \tag{9}$$

where r is the present radius to the ascending vehicle, r_T is the radius to the target position at the point of rendezvous, ϕ_a is the range angle from r to r_T, and m_a is the slope of the target orbit at the rendezvous point.

The determination of $\mathbf{V}_c$ for each position of the ascending vehicle is an iterative process, wherein the range angle ϕ_a to the rendezvous point is determined in the following steps (Fig. 5):

1) Assume a value of ϕ_a, the geocentric range angle from the vehicle's present position to the rendezvous point. (A starting value at rocket ignition is stored as a function of target lead angle.)

2) Compute the geocentric radius and slope of the target orbit at the assumed rendezvous point.

3) Compute the components of characteristic velocity.

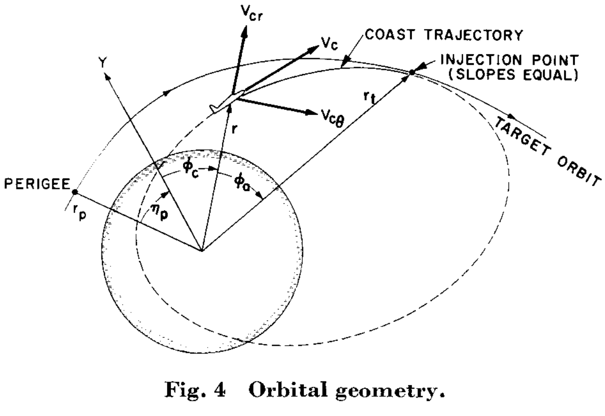

Fig. 4 Orbital geometry.

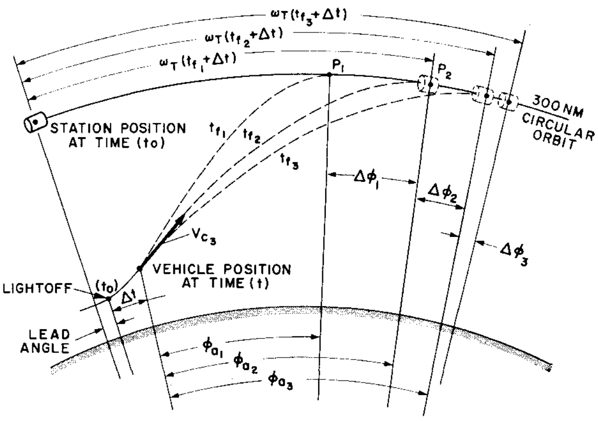

Fig. 5 Range angle iteration.

4) Compute the flight time for the assumed ascent.

5) Compute the target location at the end of the total elapsed time (Kepler iteration).

6) Compute the difference (error) in the geocentric range angle between the assumed rendezvous point in step 1 and the target location in step 5.

7) Use the target location in step 5 as the new assumed range angle to the rendezvous point, and repeat steps 2–6 until the residual error in step 6 is acceptable.

When the number of iterations goes to 3, the iteration is accelerated by assuming an exponential fit to the three points and using the asymptotic value as the next trial for ϕ_a. All subsequent iterations use the previous cycle converged value as a starting value. Experience with a great many simulated ascents indicates that this procedure generally keeps the number of iterations per computing cycle below six even at the beginning of the ascent.

The flight time for each value of ϕ_a is computed as follows:

$$t_f = \frac{1}{c}\int_0^{\phi_a}\frac{d\theta}{Q^2(\theta)} \tag{10}$$

where

$$\frac{1}{r(\theta)} = Q(\theta) = \frac{\mu}{c^2} + \left\{\frac{1}{r} - \frac{\mu}{c^2}\right\}\cos\theta - \frac{V_{cr}}{c}\sin\theta \tag{11}$$

and

$$c = rV_{ct} \tag{12}$$

A 50-strip Simpson's rule is used to perform the numerical integration.

Only X and Y components of $\mathbf{r}$ are used in the computations of $\mathbf{V}_c$. This gives satisfactory pitch guidance during the early portions of the explicit guidance period and ceases to be an approximation as the vehicle enters the target orbit plane before thrust cutoff.

The pitch steering used in the digital-computer simulation in this study is a commanded rate of change of a. A "required" function of a, $F_R(a)$ an "actual" function of a, $F_A(a)$ are defined. The steering loop commands a change in $F_A(a)$ sufficient to bring it into equality with $F_R(a)$ in one computing cycle Δt:

$$F_R(a) = F_A(a) + [dF_A(a)/da]\Delta a \tag{13}$$

The steering equation is

$$\dot{a} = \frac{\Delta a}{\Delta t} = \frac{F_R(a) - F_A(a)}{dF_A(a)/da}\left\{\frac{1}{\Delta t}\right\} \tag{14}$$

A suitable limit on $\dot{a}$ is included in the computer program to approximate a practical vehicle capability. The vehicle attitude control is assumed to have a very fast response, so that the actual value of a is assumed equal to the commanded value.

When the vehicle is following the velocity-altitude profile, $F_R(a)$ and $F_A(a)$ are defined as follows:

$$F_R(a) = dU_r/dt = [df(h)/dh]\,(dh/dt) \tag{15}$$

$$F_A(a) = d|\mathbf{U}|/dt \tag{16}$$

When these are substituted in Eq. (14) and it is noted that

$$dh/dt = d|\mathbf{r}|/dt = (X^2 + Y^2 + Z^2)^{1/2}(x\dot{x} + y\dot{y} + z\dot{z}) \tag{17}$$

the steering equation becomes

$$\dot{a} = \frac{[df(h)/dh](d|\mathbf{r}|/dt) - (d|\mathbf{U}|/dt)}{d[d\mathbf{U}/dt]/da}\,\frac{1}{\Delta t} \tag{18}$$

where $d|\mathbf{U}|/dt$ is the component of $d\mathbf{U}/dt$ along $\mathbf{U}$.

When the vehicle is being guided explicitly, $F_R(a)$ is the direction of V_g, defined by

$$\theta_g = \tan^{-1}[(V_{cy} - \dot{Y})/(V_{cx} - \dot{X})] \tag{19}$$

where

$$\mathbf{V}_g = \mathbf{V}_c - \mathbf{V} \tag{20}$$

and V_{cx} and V_{cy} are the rectangular components of $\mathbf{V}_c$.

$F_A(a)$ is the direction θ_F of the total measured force $\mathbf{F}$:

$$\mathbf{F} = \mathbf{T} + \mathbf{L} + \mathbf{D} = \Sigma\mathbf{F} - (\mu\mathbf{r}/|\mathbf{r}|^3) \tag{21}$$

$\Sigma\mathbf{F}$ is the total force. The direction of $\mathbf{F}$ is

$$\theta_F = \tan^{-1}\left[\frac{\Sigma F_y - (\mu/r^3)Y}{\Sigma F_x - (\mu/r^3)X}\right] \tag{22}$$

When $\theta_F = \theta_g$, the vehicle enters the explicit guidance mode. A rate term is added to the steering equation for stability in this mode:

$$\dot{a} = \frac{\theta_g - \theta_F}{d\theta_F/da}\left\{\frac{1}{\Delta t}\right\} + \dot{\theta}_g \tag{23}$$

where $\dot{\theta}_g$ is generated in the digital computer by

$$\dot{\theta}_g = (\theta_{gi} - \theta_{g(i-1)})/\Delta t \tag{24}$$

Out-of-Plane Guidance

The out-of-plane guidance must 1) guide the vehicle into the ascent plane ($Z = 0$) smoothly to permit accurate alignment before thrust cutoff; 2) accept a wide range of initial conditions, $Z = Z_0$ and $\dot{Z} = \dot{Z}_0$ at rocket ignition, $t = 0$; and 3) minimize maneuvers low in the atmosphere which increase propellant consumption.

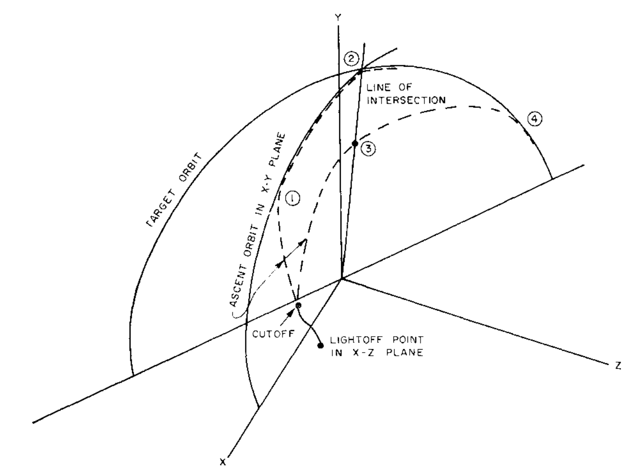

Fig. 6 Ascent in a slant plane to allow greater distance of light-off point from the target orbit plane.

The first two requirements are satisfied when the vehicle follows a "principal turn" defined by

$$\dot{Z}_R = -K_1|Z|^n \tag{25}$$

where $\dot{Z}_R$ is the required value of the component of velocity in the Z direction. The sign is taken so that the direction of $\dot{Z}_R$ is always toward the X, Y plane regardless of the sign of Z.

It is shown in Ref. 5 that the value of $n = \frac{1}{2}$ gives an efficient principal turn. It is faster than proportional navigation ($n = 1$) and has the property that, when the vehicle is following the path defined by (25), the acceleration $\ddot{Z}$ in the Z direction is a constant. (Incidentally, when $n = \frac{1}{2}$, the path lends itself to rapid graphical analysis using $\dot{Z}$ and t as coordinates.)

The third requirement, that maneuvers be minimized near $t = 0$, is met by constraining the vehicle to continue for a time along the inertial direction ($\dot{Z}_0/\dot{X}_0$) that it possessed at rocket ignition.

The out-of-plane guidance rule operates as follows. When there is an initial displacement Z_0, a check is made to insure that the component $\dot{Z}_0$ of the initial velocity

$$\mathbf{V}_0 = \mathbf{i}\dot{X}_0 + \mathbf{j}\dot{Y}_0 + \mathbf{k}\dot{Z}_0 \tag{26}$$

which is directed toward the X, Y plane, is such that the vehicle can turn into the plane before thrust cutoff. The vehicle is then commanded initially to follow the constant inertial direction defined by

$$\dot{Z}_R = \dot{X}(\dot{Z}_0/\dot{X}_0) \tag{27}$$

At each value of vehicle position, the guidance system computes a principal turn compatible with the present position and velocity and computes the value of K_1 for this principal turn:

$$K_1 = \dot{Z}/|Z|^n \tag{28}$$

Next, the time t_a is computed at which the vehicle would arrive at a point Z_a arbitrarily close to the X, Y plane if it followed this principal turn starting from its present position and velocity:

$$\int_t^{t_a} dt = \int_Z^{Z_a} -\frac{Z^n}{K_1}\,dZ \tag{29}$$

which gives

$$t_a - t = -\frac{1}{K_1}\left[\frac{Z^{1-n}}{1-n}\right]_Z^{Z_a} \tag{30}$$

Neglecting Z_a compared to Z, the time of arrival in the X, Y plane will be

$$t_a - t = 2(Z)^{1/2}/K_1 \tag{31}$$

As the vehicle proceeds, this computed value of t_a will become smaller. As soon as

$$t_a \leq t_2 \tag{32}$$

where t_2 is a specified time that allows a period for alignment before the anticipated time of thrust cutoff, the guidance is transferred from the constant inertial direction (27) to the principal turn defined by (25) and (28).

The yaw steering signal is a commanded roll rate

$$\dot{\varphi}_R = K_2\epsilon_v + K_3\dot{\epsilon}_v \tag{33}$$

where

$$\dot{\epsilon}_v = \dot{Z}_R - \dot{Z} \tag{34}$$

and $\dot{Z}_R$ is the required value of $\dot{Z}$.

Values of K_2 and K_3 in (33) are chosen to make the steering loop well damped when the aircraft is following the turn defined by (25). The combination of the principal path and steering loop equations gives a third-order nonlinear equation for Z; the stability becomes small as Z decreases. It is desirable, therefore, to use a "fine steering mode" for small values of Z in which K_1 is reduced, or in which $\dot{Z}_R$ is no longer a function of Z. In this study, initial values of Z were as great as 80 miles, and the fine steering mode became operative when Z decreased to a few hundred feet. It is also desirable, as the altitude increases, aerodynamic force decreases, and **U** loses its significance, to direct the centerline of the vehicle along **V** rather than **U**.

The preceding paragraphs describe the out-of-plane guidance when the aircraft can use the target orbit plane as the ascent plane (the X, Y plane). When the light-off point is so far from the target orbit plane that the aircraft cannot turn and align itself in this plane before thrust cutoff, the ascent is made in a slant plane in the following manner. The guidance system selects an ascent plane that intersects the target orbit plane and that meets these criteria:

1) The ascent plane is sufficiently close to the light-off point so that the vehicle can turn into it and align itself before thrust cutoff. For the vehicle in the subsequent example, this distance would be 80 miles or less.

2) The line of intersection of the ascent plane and the target orbit plane should be at a range angle such that the velocity required for the plane change will be near minimum. This will tend to be in the neighborhood of 90°.

The X and Y axes are still defined to be in the ascent plane. (If the line of intersection is 90° from the light-off point, it will be the Y axis.) Next a fictitious orbit is defined in the ascent plane by rotating the actual target orbit about the line of intersection into the ascent plane (Fig. 6). A pseudo-target is now defined to be moving in the fictitious orbit with a phase such that it meets the actual target at each crossing of the line of intersection, twice during each orbital revolution.

The guidance scheme operates to rendezvous with the psuedo-target, just as if the ascent plane were the actual target orbit plane. If rendezvous is achieved with the pseudo-target before the intersection of the two planes is reached, the vehicle injects into the orbit of the fictitious target in the ascent plane. It then coasts in this orbit coincident with the psuedo-target to the intersection, where it meets the actual target. The vehicle turns into the target orbit plane by executing a terminal rendezvous maneuver. If the vehicle reaches the line of intersection of the planes during its coasting ascent before reaching the psuedo-target, it turns into the target plane and completes the rendezvous directly with the actual target. Both of these possible situations are shown in Fig. 6.

This type of ascent uses the flexibility of the lifting turn to the full extent of the vehicle's capability. In particular, this maintains the ability to accommodate variations in position, heading, and time at rocket ignition.

Results and Conclusions

Rendezvous wherein the ascending vehicle was guided by the system described herein were simulated on a digital computer for a number of initial conditions. The objectives were, first, to test the feasibility of this rule for use in vehicle-borne computers; and second, to compare the fuel consumption to that reported in Ref. 5 for a vehicle that covers the same range angle during the ascent but for which the range angle is predetermined (not a rendezvous). The shape of the ascents are not, in general, the same for the two cases. The third and principal objective was to investigate the amount of flexibility which is possible in the conditions at rocket ignition, that is, the space-time launch "window" or "footprint" from which a vehicle can successfully rendezvous with a specified orbital target.

The vehicle characteristics assumed in the simulation were such as might reasonably be found in a lifting ascent vehicle: maximum lift/drag in the neighborhood of 4; altitude and velocity at rocket ignition in the neighborhood of 100,000 ft and 6000 fps. The magnitude of the velocity **U** relative to the air mass at rocket ignition was held constant for all ascents. The components of **U** were varied, depending upon the direction of the ascent relative to the rotating air mass and upon the desired value of $\dot{Z}_0$. Rocket thrust was assumed constant throughout a thrusting period of approximately 360 sec. For most of the simulations, the target was assumed to be in a 300-naut-mile polar, circular orbit.

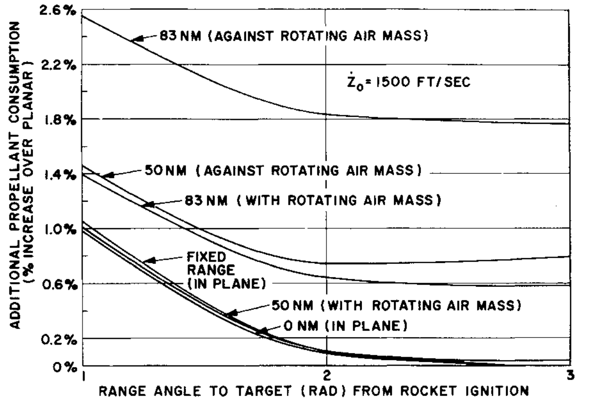

Fig. 7 Additional propellant consumption as a function of range angle, and initial distance from ascent plane. Planar, 3-rad-range-angle case is used as reference.

The conditions at rocket ignition were varied over the range of vehicle capability. The lead angle from the ascending vehicle to the target at rocket ignition was varied so that the range angle covered during ascent varied between approximately 0.9 and 3 rad. The increase in fuel consumption became significant at shorter range angles, and range angles longer than 180° cannot be achieved with a single thrusting period. The initial distance from the target orbit plane Z_0 was varied from zero to 500,000 ft. The allowable values of heading or component of initial velocity directed toward the target plane $\dot{Z}_0$ varied with Z_0; a range of at least 1000 fps is possible at any value of Z_0.

Ascent rendezvous were completed successfully over this range of initial conditions. The accuracy of arrival at the target was good; errors resulting from the computational scheme and from the idealized steering loops totaled between a few feet and a few hundred feet. The number of iterations to determine characteristic velocity was generally less than six at any vehicle position. The guidance scheme appears suitable for vehicle-borne computers.

For the same value of range angle traversed during ascent, the shape of the rendezvous ascent was generally different from the path when the range to the target was predetermined. For example, the vehicle generally entered the explicit guidance mode earlier in a rendezvous ascent The fuel, however, which was required for a rendezvous ascent to a target at a given range angle, was generally within 0.1% of the fuel required to traverse the same range angle, predetermined. Figure 7 shows fuel requirement as a function of range angle for various values of Z_0 and $\dot{Z}_0$ and includes data from Ref. 5 for targets at predetermined range angles for comparison. Figure 8 shows fuel as a function of heading ($\dot{Z}_0$) for various values of Z_0 and range angle, to show the variation in heading which is permissable at rocket ignition.

The investigation of launch window indicates that, for a target in a 300-mile circular orbit, when the geocentric lead angle between the target and the ascending vehicle at rocket ignition is 0.248 rad, the vehicle will traverse 0.8 rad to rendezvous; when the lead angle is 0.04 rad at ignition, the vehicle will traverse approximately 3.3 rad to rendezvous. Furthermore, for a given lead angle at rocket ignition, the range angle to rendezvous does not vary significantly (<0.01

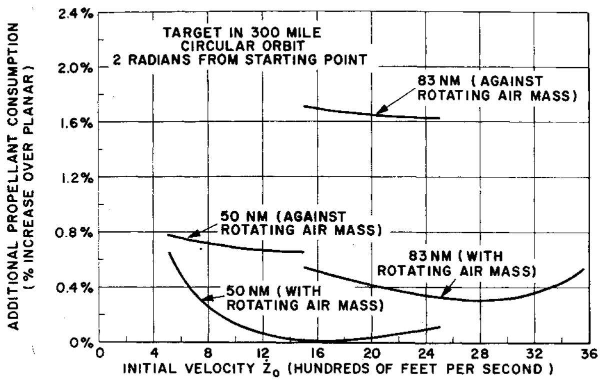

Fig. 8 Additional propellant consumption as a function of component of initial velocity $\dot{Z}_0$ toward ascent plane.

rad) as Z_0 and $\dot{Z}_0$ vary. This means that, if rocket ignition were constrained to occur at a fixed position near the orbital plane, the allowable time variation or launch window would be equal to the time for the orbital target to traverse a geocentric angle of (0.248 − 0.04) rad, or 3.16 min.

The flexibility in conditions at rocket ignition which is permitted the ascending vehicle is much broader than is indicated by this single number. The vehicle will be in a hypersonic cruise, with a velocity component parallel to that of the target equal to approximately one-fifth that of the target velocity, so that the allowable rocket ignition time is extended by a factor of approximately $\frac{6}{5}$ to approximately 225 sec. More important, the vehicle can be following any of a very large number of possible paths, so long as it is sufficiently close to the orbital plane, with a reasonable heading, during at least a part of the time interval. The values of out-of-plane distance Z_0 and heading $\dot{Z}_0$ can vary considerably.

The following concept may assist in the visualization of the spacetime region over which rocket ignition is allowed to take place, for successful rendezvous. Consider that the target in orbit casts a shadow that covers an area on the earth's surface surrounding the ground track of the target orbit. The shadow travels along the ground track, over the earth's surface, at the speed of the orbital target and somewhat ahead of it. (Figure 9 illustrates this.) The ascending vehicle can begin its rocket-powered ascent at any instant at which the geocentric position vector to the ascending vehicle passes through the earth's surface within the shadow of the orbital target, so long as the vehicle's heading ($\dot{Z}_0$) is reasonable. For the vehicle configuration used in the simulation in this study, the shadow is approximately a rectangle along the target orbit ground track and extending approximately 80 miles on each side of it.

In Fig. 9, the ascending vehicle is in a hypersonic cruise that places it approximately 70 miles from the target orbit plane when the leading edge of the target shadow overtakes it; the vehicle has a constant component of velocity toward the plane ($\dot{Z}_0$) of 1500 fps. Rocket ignition can take place at any time during the next 240 sec, until the trailing edge of the target shadow leaves the vehicle. At the end of this time it will be approximately 10 miles from the target orbit plane. This is only one of a very large number of hypersonic cruise paths from which the vehicle can begin its rocket ascent.

Different guidance constraints might be used to define the coasting ascent ellipse; these would be implemented by expressions for V_c different from (8) and (9). The guidance scheme would still be able to accept changing values of position and heading at rocket ignition. One such scheme will be considered briefly here, in which V_c is defined so that the vehicle traverses a fixed range angle (nominally fixed time of flight) from rocket ignition to rendezvous. For comparison, the guidance scheme in which V_c is defined by (8) and (9) will be designated system 1, and the scheme that traverses a predetermined range angle system 2. It will be assumed that system 2 is identical to system 1 insofar as out-of-plane guidance, use of velocity-altitude profile, and atmospheric cross-product steering for ascent are concerned.

Rocket ignition in system 2 will normally occur when the target lead angle is such that the vehicle will arrive at the rendezvous point at the same time as the target. System 2, like system 1, can be thought of as having associated with it a traveling "shadow" cast by the target which defines the allowable region over which rocket ignition can take place at any instant, or the allowable time interval over which ignition can take place at any given position. The variation is obtained by recomputing V_c so that the vehicle will traverse the same range angle but will arrive at rendezvous with a different slope; this will cause a variation in time of free flight.

The total velocity required varies as the slope at arrival is varied. Figure 10 is a plot of the sum of the velocity at cutoff and at injection into the target orbit, as a function of the time of free flight, as slope at arrival is varied, for two values of range angle. The target is in a 300-mile circular orbit. Minimum total velocity is required when the slope at arrival is nearly equal to that of the target orbit; the velocity requirement increases as the slope moves away from this value.

Figure 11 is a plot of the additional velocity increment required as the allowable variation in rocket ignition time is increased for systems 1 and 2. The data for system 1 are taken from Fig. 7, converted to feet per second, and the data for system 2 from Fig. 10. The reference values of velocity are the minimum values that could bring the vehicle to meet the target. For an allowable variation in time of rocket ignition of 3.1 min, the velocity increment is 50 fps for system 1;

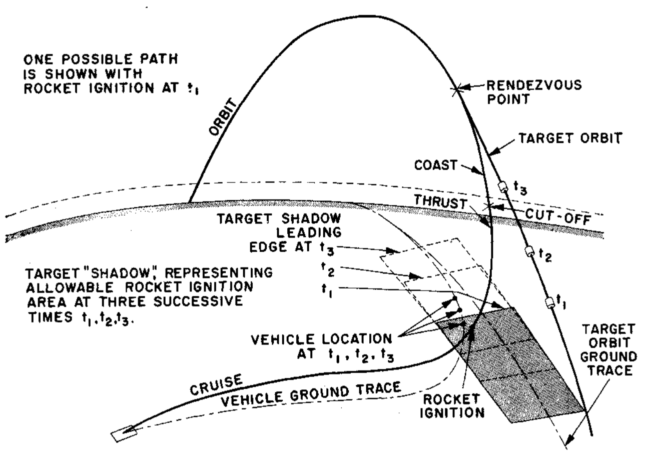

Fig. 9 Target "shadow" representing allowable rocket ignition area at three successive times t_1, t_2, t_3.

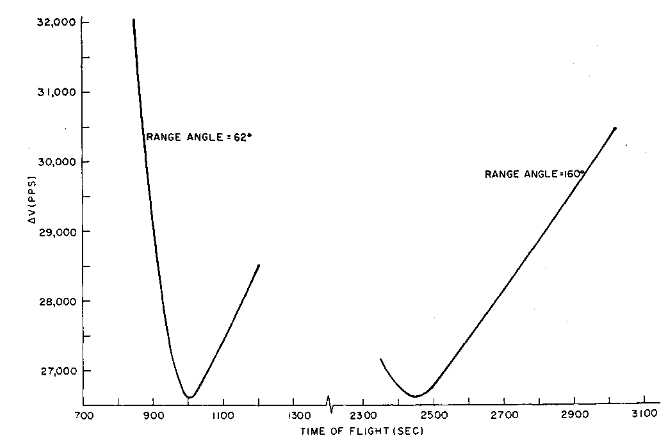

Fig. 10 Sum of cutoff plus orbital injection velocities as a function of time of free flight as slope at injection is varied.

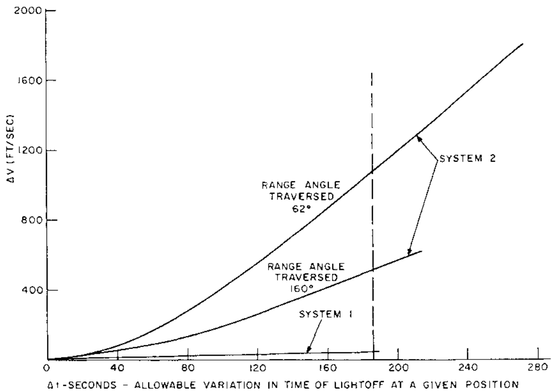

Fig. 11 Velocity increment required for variation in allowable time of light-off at a given position.

for system 2 it is 1080 fps if a range angle of 60° is used and 510 fps if the range angle is 160°. The 50 fps that would allow a variation of 3.1 min in ignition time at any given position for system 1 would allow only 30 to 40 sec for system 2, depending upon the predetermined range angle. In other words, when system 2 is used for the same velocity increment, the traveling "shadow" cast by the orbital target is significantly shorter than the shadow cast by the same target when system 1 is used, and which is shown in Fig. 9.

In the final analysis, the guidance method for each future lifting ascent vehicle will be developed to include those features that will satisfy the particular requirements that are generated by the mission characteristics. This paper has described a method that is believed to offer a good compromise among flexibility, fuel, and time for a lifting vehicle that has a hypersonic cruise capability, and when a direct ascent to rendezvous is desired. One of the possible variations has been discussed briefly: to show how particular elements of the guidance scheme can be altered if necessary to fit changes in requirements.

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